What information can we obtain from the yield ratio π^-/π^+ in heavy-ion collisions ?

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Abstract

The recently reported data on the yield ratio π^-/π^+ in central rapidity region of heavy-ion collisions are analyzed by theoretical formula which accounts for Coulomb interaction between central charged fragment (CCF) consisting of nearly stopped nucleons with effective charge $Z_{\rm eff}$ and charged pions produced in the same region of the phase space. The Coulomb wave function method is used instead of the usual Gamow factor in order to account for the finite production range of pions, β. For Gaussian shape of the pion production sources it results in a quasi-scaling in β and $Z_{\rm eff}$ which makes determination of parameters β and $Z_{\rm eff}$ from the existing experimental data difficult. Only sufficiently accurate data taken in the extreme small m_T-m_π region, where this quasi-scaling is broken, could be used for this purpose.

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The ratios of pionic yields π^-/π^+ in the central rapidity region in high energy heavy-ion collisions have been recently reported by E866 [1] and by NA44 [2] Collaborations. In both cases a strong excess of yield of π^- over that of π^+ in the low transverse mass region has been found. Although its theoretical origin is still point of debate, the most obvious reason to be checked at first instance is the effect of the Coulomb final state interaction taking place between charged pions and central charged fragment (CCF) consisting of nearly stopped nucleons with effective charge $Z_{\rm eff}$ and produced in the central rapidity region [3], cf. Fig. 1. In the approximation neglecting

the possible effect of the finite range of the production of pions as seen, for example, in Bose-Einstein correlation experiments, the observed results are given by the ratio of Gamow factors only [3]:

$$N^{\pi^+}(\mathrm{p_{rel}})/N^{\pi^-}(\mathrm{p_{rel}}) = G(\eta)/G(-\eta), \tag{1}$$

where Gamow factor $G(\eta)$ is defined as $G(\eta) = 2\pi\eta\ /(\exp(2\pi\eta) - 1)$ and $\eta = Z_{\mathrm{eff}}\ m_\pi\alpha/\mathrm{p_{rel}}$. Here α, m_π and $\mathbf{p}_{\mathrm{rel}}$ are the fine structure constant, mass of π meson and its relative momentum in the two-body system (π-CCF), respectively. The energies considered here are high enough to neglect the influences of charged spectators produced in the fragmentation regions, cf. Fig. 1.

Such problems have been already considered before but either at much lower energies [4, 5] or with the use of Gamow factor only [6]. In this work we shall analyse high energy data and use the Coulomb wave function method as developed in [7] instead of simple Gamow factors to describe the final state Coulomb interactions. This allows us to account for the finite size of the emitting source (or, equivalently, for the finiteness of the pion production range) characterized by parameter β. The question we are interested in is: can one obtain from the observed yields of π^-/π^+ production mentioned before any valuable information about *both* the effective charge of CCF, Z_{eff}, and the range of interaction parameter β ? We derive theoretical formula for the pion production yield based on the Coulomb wave function convoluted with some pionic source function $\rho(\mathrm{r})$ and apply it to the analysis of recent experimental data on Au+Au collisions at 11.0 GeV/nucleon [1] and on Pb+Pb Collisions at 158 GeV/nucleon [2]. The emerging valley-like structures in the map of the χ^2-values made in β-Z_{eff} parameter space is shown to be connected with the quasi-scaling behaviour of integrals of the square of Coulomb wave function convoluted with pion production source function [8]. This finding shows that the expected determination of parameters β and Z_{eff} from the existing experimental data is difficult if not impossible.

Let us consider Coulomb interaction between π^+ with lab rapidity y_π and CCF of some effective mass M_{eff} and with rapidity Y_{CM} (which we assume to be equal to the cms rapidity of the colliding system). In the rest frame of CCF the π^+ momentum $\mathbf{p}_\pi$ is given by

$$\mathbf{p}_\pi\ \equiv\ (\ \mathrm{p}_L, = (\ m_T \sinh(y_\pi - Y_{\mathrm{CM}})\ ,\ \mathbf{p}_T\ ) \tag{2}$$

where $m_T = \sqrt{m_\pi^2 + \mathbf{p}_T^2}$ and $\mathbf{p}_T$ is its transverse momentum. In this frame the Schrödinger equation for the relative motion of the two-body system

(π^+-CCF) is given by

$$\left[\ \frac{\widehat{\mathbf{p}}_r^2}{2\mu} + \frac{Z_{\rm eff}\ e^2}{r}\ \right]\psi_r(\mathbf{r}) = E_r\psi_r(\mathbf{r}), \tag{3}$$

where $\mathbf{r}$ is the relative coordinate of π^+ in the rest frame of the CCF, $\widehat{\mathbf{p}}_r \equiv -i\hbar\nabla_r$ and μ is the reduced mass of our two-body system: $\mu = m_\pi M_{\rm eff}/(M_{\rm eff}+m_\pi) \approx m_\pi$ [9]. One can find that the wave function ψ_r is given by the following confluent hypergeometric function [7, 10]:

$$\begin{aligned}
&\psi_r(\mathbf{p}_r,\mathbf{r}) = \\
&\Gamma(1+i\eta)e^{-\pi\eta/2}e^{i\mathbf{p}_r\cdot\mathbf{r}}F(\,-i\eta,1,i\ (\mathrm{p}_r r-\mathbf{p}_r\cdot\mathbf{r}))
\end{aligned} \tag{4}$$

where $\mathbf{p}_r = M_{\rm eff}\ \mathbf{p}_\pi/(M_{\rm eff}+m_\pi)$. Convoluting now eq.(4) with some source function $\rho(\mathbf{r})$ (i.e., distorting the Coulomb wave function accordingly in order to account for the finite production range of pions), we can now calculate single particle spectra of produced π^+'s:

$$\begin{aligned}
&N^{\pi^+}(\mathbf{p}_r;\eta_+,\beta) = \int d^3\mathbf{r}\ \rho(\mathbf{r})\ \left|\psi_r(\mathbf{p}_r,\mathbf{r})\right|^2 = \\
&\quad G(\eta_+)\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{(-i)^n(i)^m}{n+m+1}A_n(\eta_+)A^*_m(\eta_+) \\
&\quad \times I_R(n,m)\ (2\ \mathrm{p}_r)^{n+m},
\end{aligned} \tag{5}$$

where $A_n(\eta_+) = \Gamma(i\eta_+ + n)/(\Gamma(i\eta_+)(n!)^2)$ and $\eta_+ = +Z_{\rm eff}\ \mu\alpha/\mathrm{p}_r$. Assuming now that $\rho(r)$ is given by Gaussian distribution: $\rho(r) = \left(\frac{1}{\sqrt{2\pi}\beta}\right)^3\exp\left(\frac{-r^2}{2\beta^2}\right)$, in which case

$$\begin{aligned}
I_R(n,m) &= 4\pi\int_0^\infty dr r^{n+m+2}\rho(r) \\
&= \frac{2}{\sqrt{\pi}}(\sqrt{2}\beta\ )^{n+m}\Gamma\left(\frac{n+m+3}{2}\right)
\end{aligned}$$

(cf. Δ_{1C} in ref.[7]), and using the following decomposition of Q^2:

$$Q^2 = \mathrm{p}_L^2 + \mathbf{p}_T^2 = \mathrm{p}_L^2 + (m_T-m_\pi)^2 + 2m_\pi(m_T-m_\pi),$$

we finally obtain that

$$\begin{aligned}
&N^{\pi^+}(m_t-m_\pi;\eta_+,\beta)\ \Big|_{\text{fixed } y_\pi} = G(\eta_+)\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{(-i)^n(i)^m}{n+m+1} \\
&\quad \frac{4}{\sqrt{\pi}}A_n(\eta_+)A^*_m(\eta_+)\Gamma\left(\frac{n+m+3}{2}\right)(\sqrt{2}\beta\ )^{n+m} \\
&\quad \times(\mathrm{p}_L^2 + (m_T-m_\pi)^2 + 2m_\pi(m_T-m_\pi)\ )^{(n+m)/2}.
\end{aligned} \tag{6}$$

For the π^- production case the corresponding N^{π^-} yield can be obtained by simply changing the sign of the Sommerfeld parameter in eq.(6): $\eta^+ \to -\eta_+ = \eta_- = -Z_{\rm eff}\ \mu\alpha/Q$. Therefore theoretical formula for the ratio of the production yields we are looking for is given by:

$$\pi^-/\pi^+ = \frac{N^{\pi^-}(m_T - m_\pi; \eta_-, \beta)}{N^{\pi^+}(m_T - m_\pi; \eta_+, \beta)}. \tag{7}$$

Using now eq.(7) we analyse data on the yield ratios π^+/π^- observed in Au+Au collisions [1] and on π^-/π^+ ratio observed in Pb+Pb collisions [2]. For Au+Au collisions data $Y_{\rm CM}$=1.58 and y_π=2.10 in eq.(2) whereas for data on Pb+Pb collisions $Y_{\rm CM}$=2.90 and for y_π we use the averaged value over rapidities of π employed in the respective data analysis. Applying the minimum χ^2-fitting method when comparing eq.(7) with experimental data one discovers (after using a lot of CPU-time) the valley-like structures in the maps of the χ^2-values made in β-$Z_{\rm eff}$ parameter space, cf. Figs. 2a and 2b for E866 and NA44 data, respectively [11]. The values of χ^2 along these valleys are almost constants and equal to $\chi^2 \approx 35/30$ and $\chi^2 \approx 52/53$ for E866 and NA44 data, respectively. The traces of the minimum χ^2-values are almost linear.

These results strongly suggest the following quasi-scaling behaviour being present in eq.(6):

$$\begin{aligned} & N^{\pi^+}(m_T - m_\pi;\ \lambda\times Z_{\rm eff}\ ,\ \lambda\times\beta) \\ & \qquad \approx\ N^{\pi^+}(m_T - m_\pi;\ Z_{\rm eff},\ \beta), \end{aligned} \tag{8}$$

where $\lambda > 0$. Fig. 3 showing the results of calculations of eq.(8) fully confirms this supposition: both for N^{π^+} and N^{π^-} and also for their ratio N^{π^-}/N^{π^+}. It means, and it is demonstrated in Fig. 4 for both sets of data analysed here, that present experimental data on yields π^-/π^+ are not able to determine parameters $(\beta, Z_{\rm eff}\ )$ uniquely. As one can see we can explain E866 data by two sets of parameters: $(\beta, Z_{\rm eff})$=(1.0 fm, 24) and (3.0 fm,72). Similarly NA44 data can be also described by two sets of parameters: $(\beta, Z_{\rm eff})$=(2.5 fm, 40) and (5.0 fm,80). Only at extreme small $m_T - m_\pi$ region, where such quasi-scaling in eq.(8) seems to be violated (cf. Fig. 5), such unique determination could be (in principle) possibly achieved (but only for sufficiently accurate data).

Summarizing: we have analyzed recent high energy data for yield ratios π^-/π^+ measured in the central rapidity regions using our theoretical

formula (7) calculated by convoluting the square of Coulomb wave function with Gaussian source function for pion production. In this way we have accounted for the distortion of Coulomb wave function caused by the finite size of the production region. With this formula we find the valley-like structures for the maps of χ^2-values in the β-$Z_{\rm eff}$ parameter space which can be attributed to the quasi-scaling behaviour of eq.(8) [8]. Results obtained by using only Gamow factor [3] do not show this property. It means, as was show in Fig. 4, that it is difficult, if not impossible, to determine parameters β and $Z_{\rm eff}$ uniquely in this case. They could be determined, if at all, only by using very accurate data obtained in the extreme small $m_T - m_\pi$ region where this quasi-scaling property in eq.(8) is broken (cf. Fig.5).

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[8] In this work we are using, as an example, a Gaussian source function $\rho(r)$ as given below. We have checked that similar scaling properties hold also for other source functions discussed in [7].

[9] In the actual calculations we evaluate μ by using $M_{\rm eff} = Z_{\rm eff}(m_p + m_n) \approx 2m_p Z_{\rm eff}$, where m_p and m_n denote the proton and neutron mass, respectively.

[10] L.I. Schiff, Quantum Mechanics, 2nd Ed. (McGraw-Hill,New York,1995), p. 117.

[11] Because asymptotic value of E866 data is equal to 0.85, in our numerical calculations we have introduced additional normalization constant equal to 0.85 when analysing these data, i.e., eq.(7) reads in this case:

$$\pi^+/\pi^- = 0.85 \times \frac{N^{\pi^+}(m_T - m_\pi;\quad \eta_+, \beta\ )}{N^{\pi^+}(m_T - m_\pi;\ \eta_-, \beta\ )}.$$

.

Figure Captions.

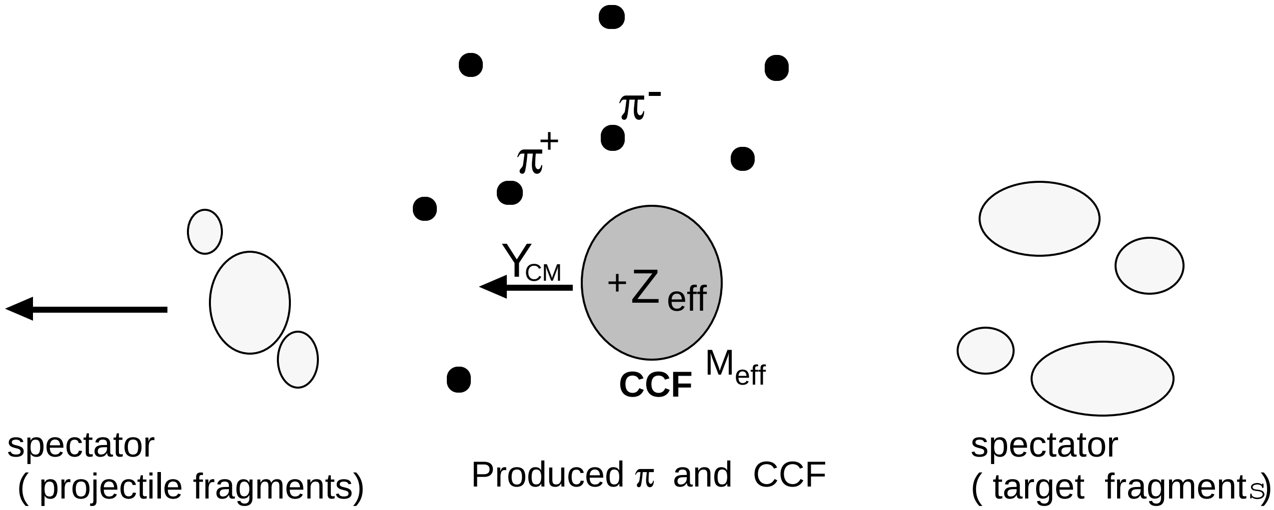
Fig. 1. The picture of collision: charged π's and CCF are produced in the central rapidity region whereas spectators populate both target and projectile fragmentation regions. The transverse energy spectra of the charged π's are affected by their Coulomb interaction with the CCF.

Fig. 2. (a) The valley-like structure exhibited by values of χ^2 (only numerators are shown, denominators, denoting the NDF's, are equal to 30 here) in the β-$Z_{\rm eff}$ parameters space for E866 data [1] fitted by using eq.(7). The minimum χ^2-value is about 35/30. (b) The same as for (a) but for NA44 data [2] (here the NDF's are equal to 53). The minimum χ^2-value is about 52/53. In both cases the $Z_{\rm eff}$ from fits using Gamow factor only are shown by the star ($Z_{\rm eff} = 7.3$ for $\chi^2 = 44/31$ for (a) and $Z_{\rm eff} = 6.9$ for $\chi^2 = 164/54$ for (b)). For E866 data eq.(7) has been renormalized accordingly [11].

Fig. 3. The quasi-scaling property of eq.(6) for: (a) N^{π^-} and N^{π^+} pion production yields and (b) for their ratio N^{π^+}/N^{π^-}. Three different (scaled by factors $\lambda = 2$ and 3) sets of parameters: $(\beta, Z_{\rm eff})$ =(1.0 fm, 24), (2.0 fm, 48) and (3.0 fm, 72) give approximately the same results (solid line). Dashed curves represent the results obtained using Gamow factor.

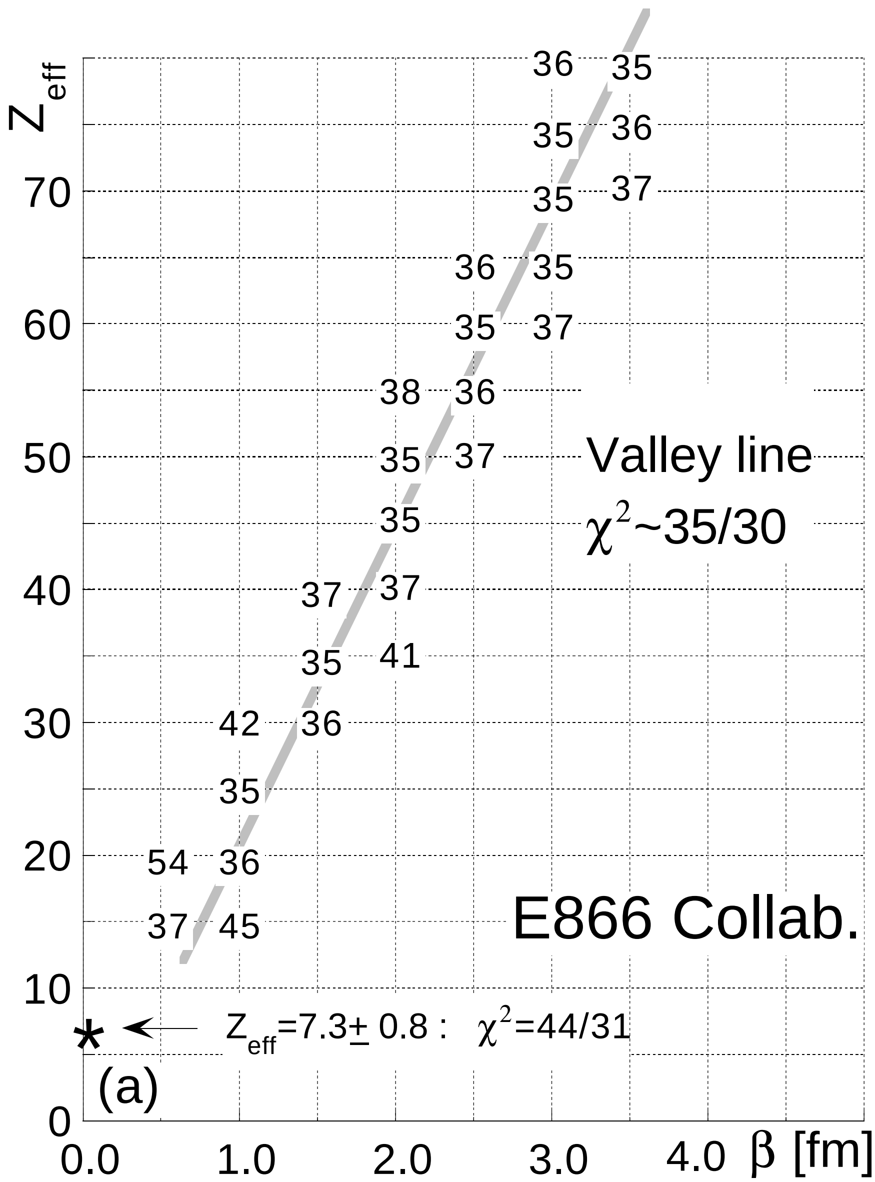
Fig. 4. (*a*) Comparison of eq.(7) with E866 data [1] using parameter sets $(\beta, Z_{\rm eff})$=(1.0 fm, 24) and (3.0 fm, 72). (*b*) The same for NA44 data [2] using parameter sets $(\beta, Z_{\rm eff})$=(2.5 fm, 40) and (5.0 fm, 80). For E866 data eq.(7) has been renormalized accordingly [11].

Fig. 5. Example of violation of the quasi-scaling property of eq.(6) for small values of the variable $(m_T - m_\pi)$ shown for N^{π^+}/N^{π^-} and compared to NA44 data [2] (here $(\beta, Z_{\rm eff})$ = (1.25fm,20), (2.5fm,40) and (5.0fm,80)).



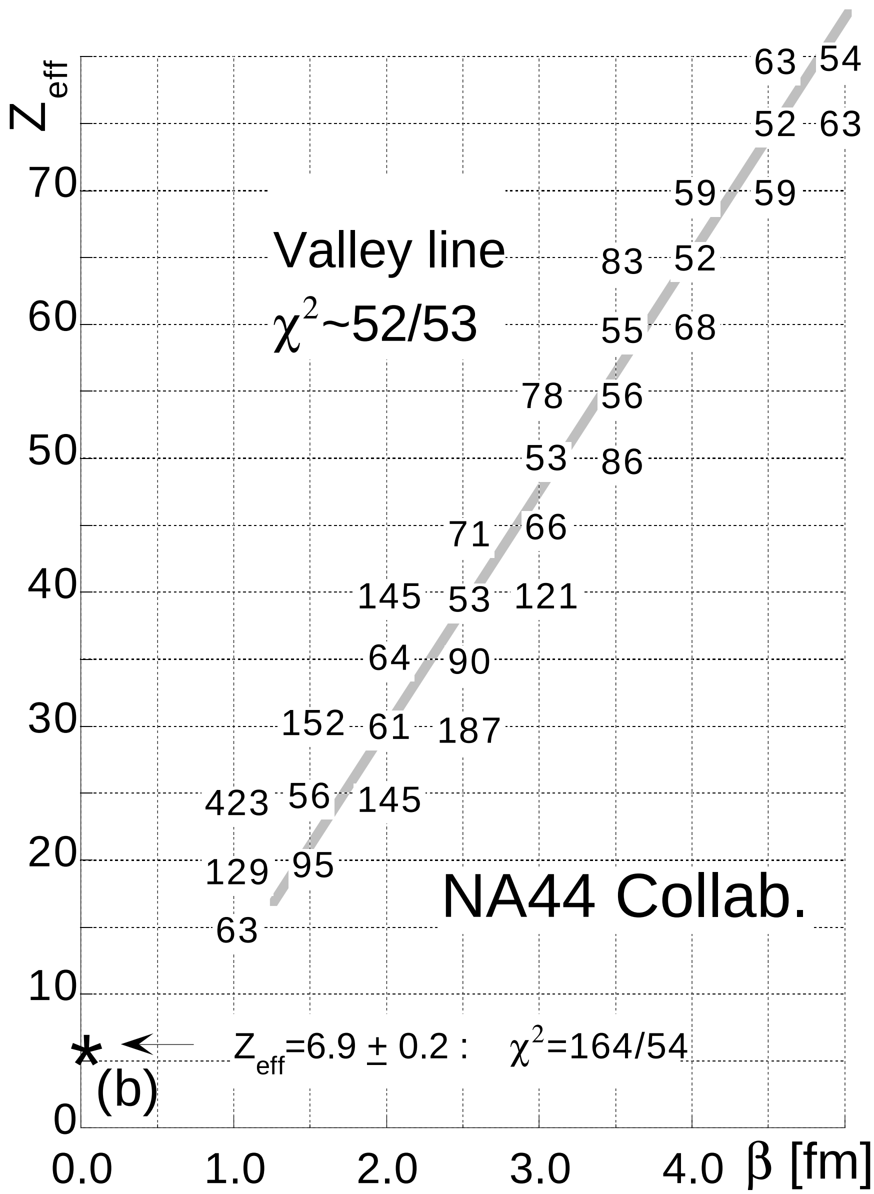
π^+
π^-
Y_{CM}
$^+Z_{eff}$
CCF
M_{eff}
spectator
(projectile fragments)
Produced π and CCF
spectator
(target fragments)

Fig. 1



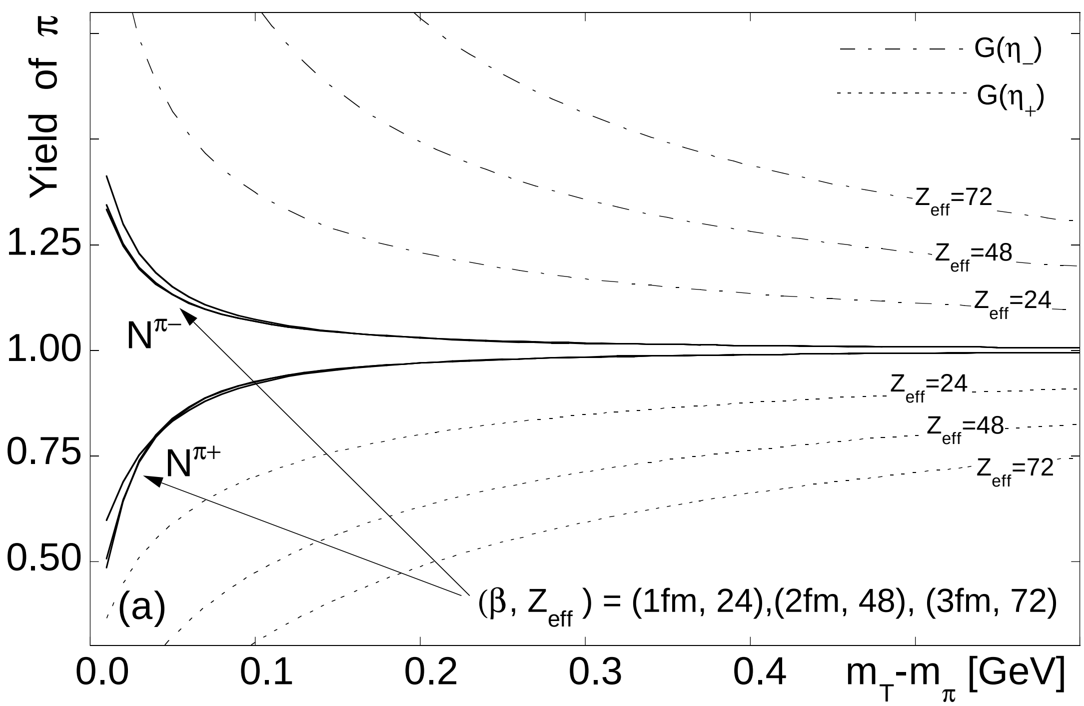
Z_{eff}
70
60
50
40
30
20
10
0
0.0
1.0
2.0
3.0
4.0
β [fm]
36
35
35
36
35
37
36
35
35
35
37
38
36
35
37
35
37
37
35
41
42
36
35
54
36
37
45
Valley line
χ^2~35/30
E866 Collab.
Z_{eff}=7.3± 0.8 : χ^2=44/31
(a)

Fig. 2



Z_{eff}
Valley line
χ^2~52/53
NA44 Collab.
Z_{eff}=6.9 ± 0.2 : χ^2=164/54
(b)
β [fm]
0.0
1.0
2.0
3.0
4.0
0
10
20
30
40
50
60
70
63
54
52
63
59
59
83
52
55
68
78
56
53
86
71
66
145
53
121
64
90
152
61
187
423
56
145
129
95
63

Fig. 2



Yield of π
1.25
1.00
0.75
0.50
0.0
0.1
0.2
0.3
0.4
m_T-m_π [GeV]
G(η_)
G(η+)
Z_{eff}=72
Z_{eff}=48
Z_{eff}=24
Z_{eff}=24
Z_{eff}=48
Z_{eff}=72
$N^{\pi-}$
$N^{\pi+}$
(a)
(β, Z_{eff}) = (1fm, 24),(2fm, 48), (3fm, 72)

Fig.3

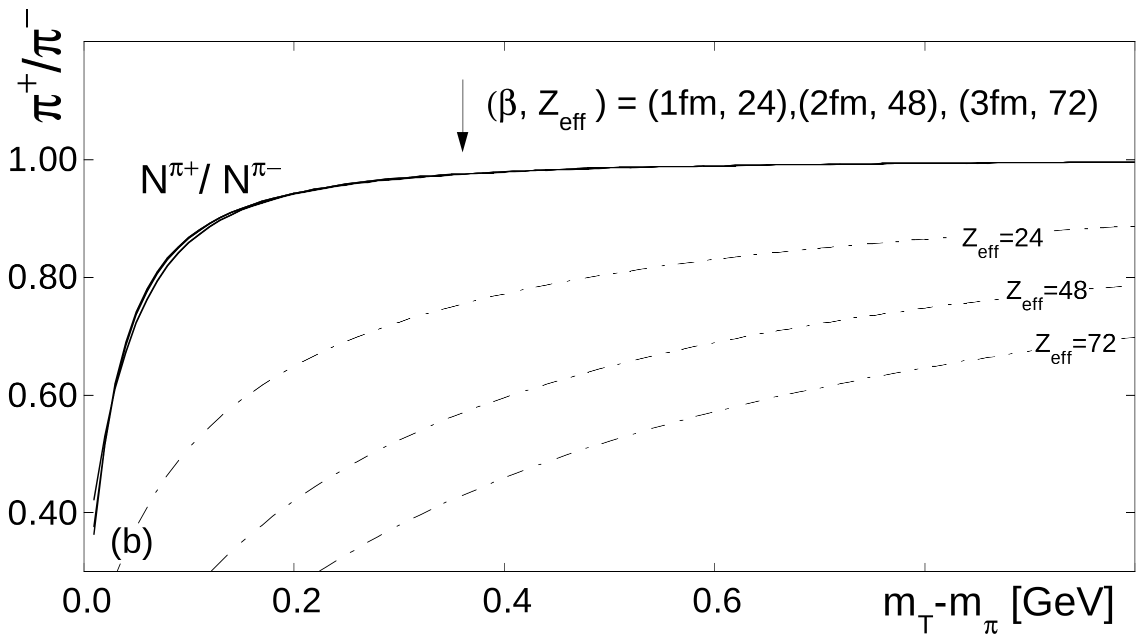
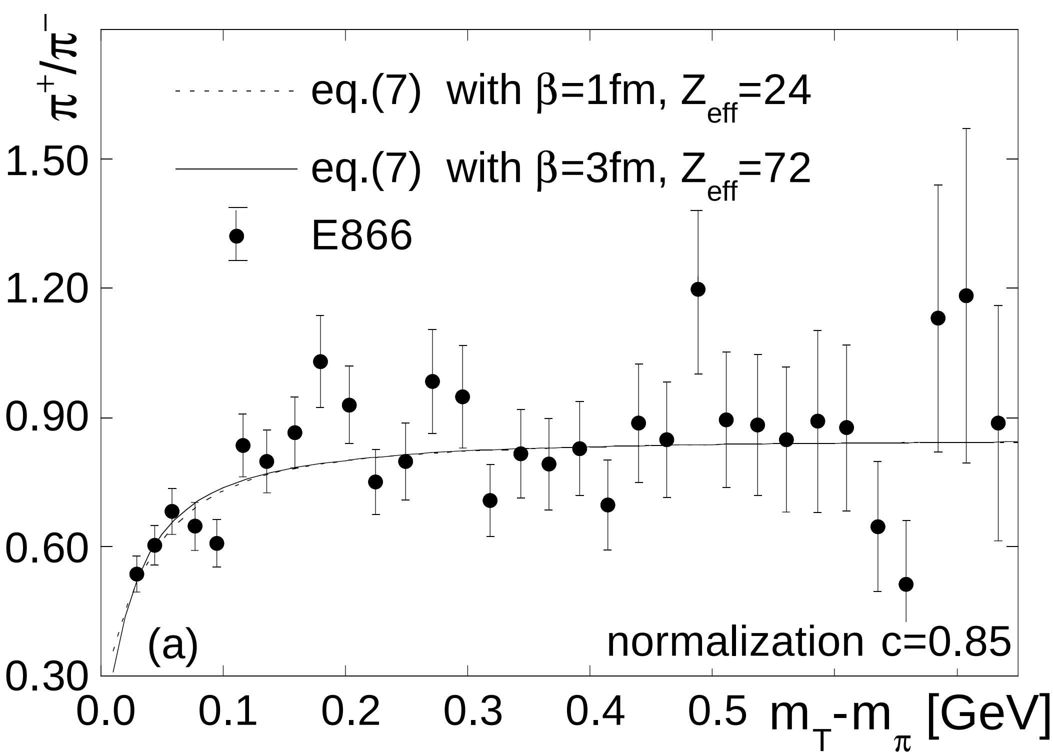
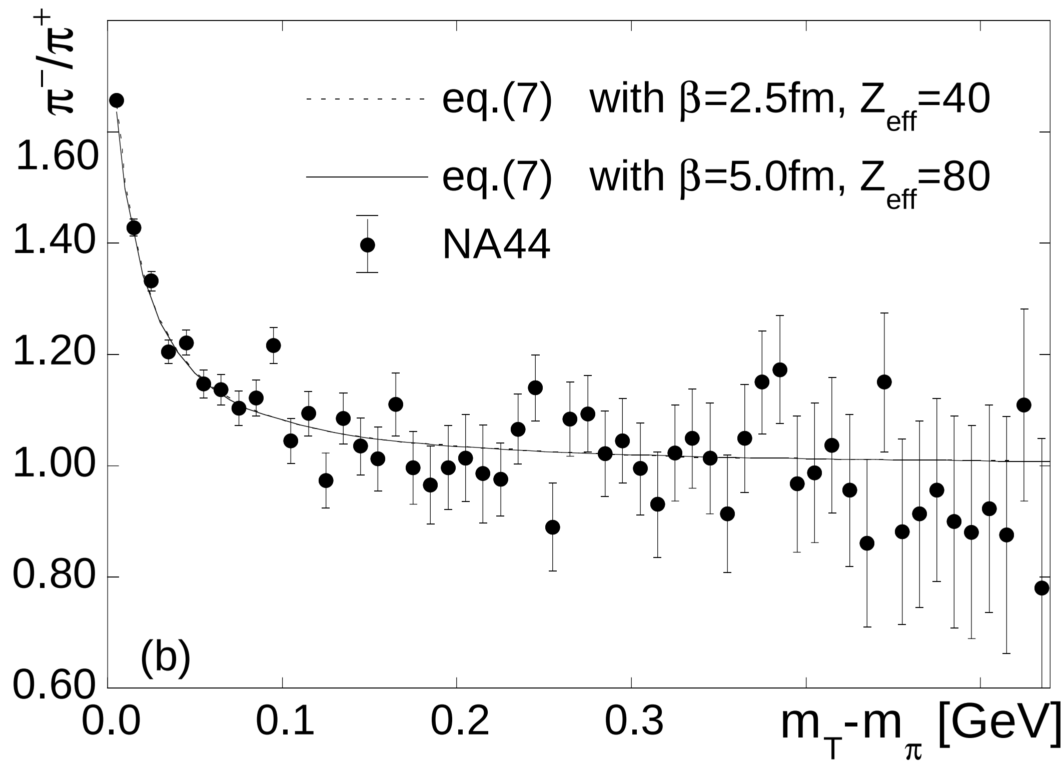

π+/π−
(β, Zeff) = (1fm, 24),(2fm, 48), (3fm, 72)
Nπ+/ Nπ−
Zeff=24
Zeff=48
Zeff=72
(b)
1.00
0.80
0.60
0.40
0.0
0.2
0.4
0.6
mT-mπ [GeV]

Fig.3



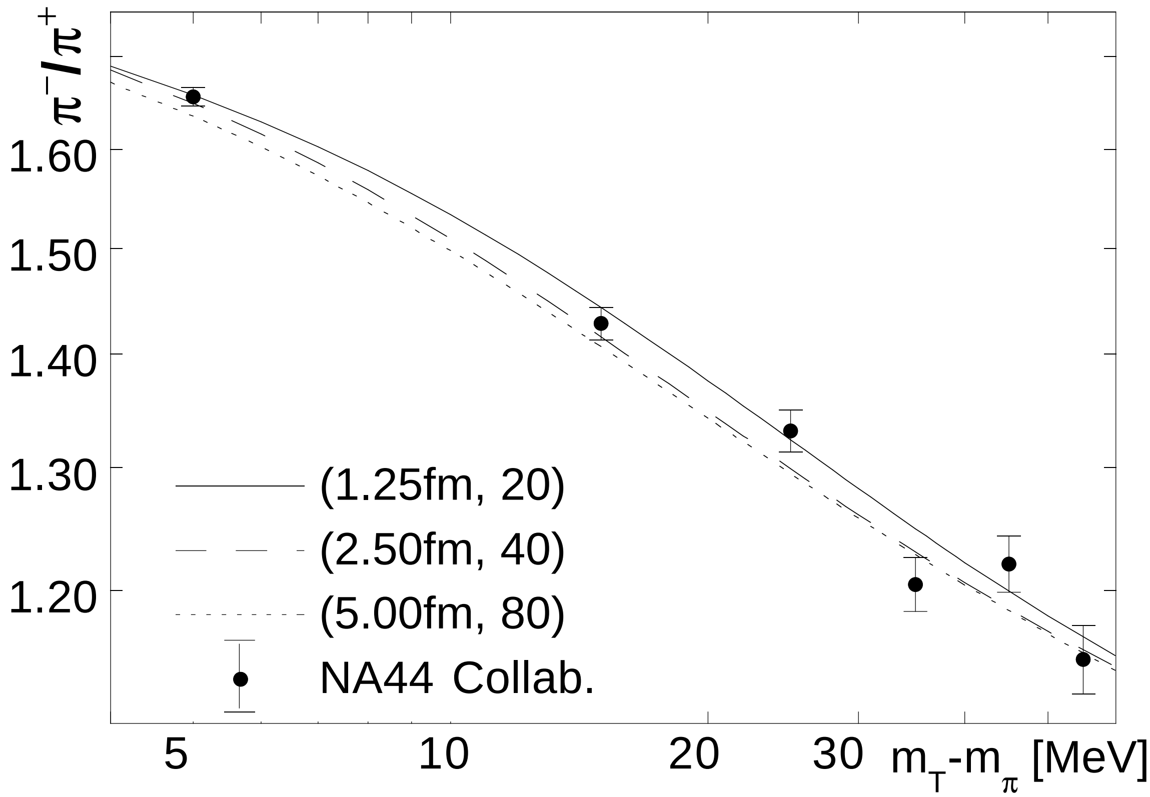
π+/π−
eq.(7) with β=1fm, Zeff=24
eq.(7) with β=3fm, Zeff=72
E866
1.50
1.20
0.90
0.60
0.30
(a)
normalization c=0.85
0.0
0.1
0.2
0.3
0.4
0.5
mT-mπ [GeV]

Fig.4



π^-/π^+
eq.(7) with β=2.5fm, Z_{eff}=40
eq.(7) with β=5.0fm, Z_{eff}=80
NA44
1.60
1.40
1.20
1.00
0.80
0.60
(b)
0.0
0.1
0.2
0.3
m_T-m_π [GeV]

Fig.4



π^-/π^+
1.60
1.50
1.40
1.30
1.20
(1.25fm, 20)
(2.50fm, 40)
(5.00fm, 80)
NA44 Collab.
5
10
20
30
m_T-m_π [MeV]

Fig.5